

# Continuous Petrov-Galerkin Method using Compactly Supported Optimal Test Functions

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**Abstract:** The numerical treatment of partial differential equations becomes a very involved problem, if multi-scale phenomena are to be dealt with. Pure continuous Galerkin discretization methods often result in unstable formulations, such as in the case of advection dominated flows whose boundary layers can then only be correctly captured using extremely fine scales. Stabilization methods such as *Streamline Upwind/Petrov-Galerkin* (SUPG), however, allow for the treatment of such problems on far more coarse scales.

We develop a continuous Petrov-Galerkin method that provides a stable and accurate solution in a norm of choice. For a given basis, optimal test perturbations are computed by solving a strictly local adjoint problem for each basis function. Linear combinations of each basis function and the corresponding perturbation serve as the problem-specific test function basis.

Currently, the perturbations are computed using a *Discontinuous Galerkin* (DG) method for  $C_0$  Finite Elements, although the method is to be extended to spline based methods.

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# SHAPE OPTIMIZATION OF AN ACOUSTIC HORN USING DIFFERENTIAL EVOLUTION AND ISOGEOMETRIC ANALYSIS

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## ABSTRACT

This work demonstrates the shape optimization of an acoustic horn to maximize impedance matching to the surrounding air<sup>1</sup>. The acoustic wave propagation was modeled by solving Helmholtz equation in Isogeometric Analysis. Differential Evolution<sup>2</sup>, which is a stochastic population-based optimization algorithm, is used to find the optimum horn bell shape. Differential Evolution is well known for its fast convergence to global optimum. A multi-objective, constrained optimization is performed to find the optimum horn shape for operating frequencies. It is shown the Differential Evolution converges fast to a global solution with excellent impedance properties for the range of desired frequencies. We found combination of Isogeometric Analysis and Differential Evolution a very promising platform for shape optimization of acoustic horn and similar devices relying on wave propagation phenomena for the following reasons

- (a) Acoustic horn analysis is very sensitive to the geometry representation in the discretized space which is presented accurately in IGA regardless of the mesh density. Isogeometric Analysis also provides higher accuracy per degrees of freedom and lower pollution error when compared with conventional Finite Element Method.
- (b) There are more than one optimum solution for acoustic horn single frequency analysis. Differential Evolution algorithm is fast converging with high success rate in finding the global solution.

We demonstrate the effectiveness of the method and compare the numerical results with those available in the literature.

- [1] E. Bangtsson, D. Noreland, M. Berggren, Shape optimization of an acoustic horn, *Computer Methods in Applied Mechanics and Engineering*, **192**, 1533-1571, 2002
- [2] R. Storn and K. Price, Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces, *Journal of Global Optimization*, **11**, 341-359, 1997.

# EXPLICIT STRUCTURAL TOPOLOGY OPTIMIZATION BASED ON ISOGEOMETRIC ANALYSIS WITH TRIMMING TECHNIQUE

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## ABSTRACT

Traditionally, structural topology optimization problems are often solved by adopting implicit geometry representation model and finite element method (FEM) on fixed mesh. This may lead to the inherent inconsistency among the geometric model, analysis model and CAD model. In order to resolve the aforementioned issues straightforwardly, in the present work, we propose an explicit structural topology optimization approach integrating the Moving Morphable Void (MMV)-based topology optimization method and Isogeometric analysis (IGA) with trimming technique. By means of the proposed approach, the present work inherits the advantages of IGA and the explicit structural topology description can be linked with CAD system seamlessly, leading to closer integration of topology design and analysis. Numerical examples provided demonstrate the effectiveness of the proposed approach.

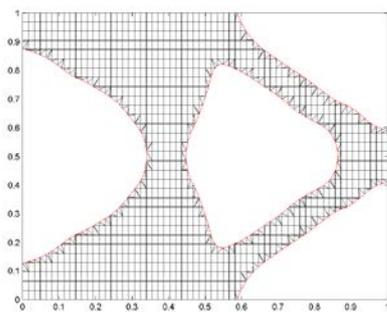


Fig. 1 Elements configuration in parametric domain.

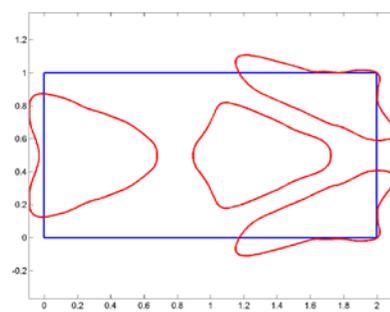


Fig. 2 CAD plot of the optimized result.

# ISOGEOMETRIC SHAPE OPTIMIZATION OF CURVILINEARLY STIFFENED AEROSTRUCTURES

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## ABSTRACT

Structural optimization requires a suitable mix of an accurate geometric description and an efficient analysis model. Isogeometric Analysis (IGA) fills this need, as it reconciles computer-aided design (CAD) with structural analysis by using spline functions as finite element bases. Since CAD uses the boundary representation (B-rep), IGA is especially suitable for analyzing structures whose geometry is easily derived from a surface, as is the case for shells. Therefore, the efficiency of IGA-based shape optimization of shells has been observed [1-2] and capable algorithms have been proposed. However, its applicability to complex structures as skin stiffened aerostuctures is not yet proven. Non-conforming interface [3] between the stiffeners and the skin is needed in order to get an attractive design space. The simplicity and the efficiency of the coupling depends on the chosen shell formulation. In addition, the shape update can be tedious when dealing with complex curved structures. Thus, we propose an approach based on Free-Form Deformation [4] to parametrize our structures which helps to modify the position of the stiffeners along the skin. Starting from numerical examples involving simple stiffened panels, the final goal is to propose a general framework for optimizing the behavior of large and complex aeronautical structures.

## REFERENCES

- [1] J. Kiendl, R. Schmidt, R. Wüchner and K.-U. Bletzinger, Isogeometric shape optimization of shells using semi-analytical sensitivity analysis and sensitivity weighting, *Comput. Methods Appl. Mech. Eng.*, **274**, 148-167, 2014.
- [2] A.P. Nagy, S.T. Ijsselmuiden and M.M. Abdalla, Isogeometric design of anisotropic shells: Optimal form and material distribution, *Comput. Methods Appl. Mech. Eng.*, **264**, 145-162, 2013.
- [3] R. Bouclier, J.-C. Passieur and M. Salaün, Local enrichment of NURBS patches using a non-intrusive coupling strategy: Geometric details, local refinement, inclusion, fracture, *Comput. Methods Appl. Mech. Eng.*, **300**, 1-26, 2016.
- [4] T.W. Sederberg, Free-Form Deformation of Solid Geometric Models, *SIGGRAPH Comput. Graph.*, **20**, 151-160, 2014.