INTRODUCTION OF ADAPTIVE W-REFINEMENT TECHNIQUE IN ISOGEOMETRIC ANALYSIS

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ABSTRACT

It is well-known that smooth tensor product splines, such as NURBS, are inherently weak in capturing rapidly varying fields and sharp local gradients. Hence, in isogeometric analysis (IGA), they perform poorly in the presence of thin layers or singularities. The prevailing idea for addressing these types of problems, which has recently been extensively explored, is providing the adaptive local refinement property by employing hierarchical splines [1,2].

In this paper, we introduce a novel adaptive w-refinement technique in isogeometric analysis based on a proposed generalization of NURBS, referred to as generalized NURBS (GNURBS), as an alternative fundamental approach for dealing with the aforementioned problems. This generalization is obtained by decoupling of the weights associated with the basis functions in geometry and field variable space. Considering the additional unknown *control weights* in the function space as design variables, we subsequently develop two different adaptivity algorithms to find these unknown variables which construct the optimal function space associated with the problem under study, while preserving the underlying geometry as well as its parameterization unchanged. These adaptive algorithms rely on a residual based a posteriori error estimator and lead to solving an unconstrained optimization problem. Having access to the full analytical sensitivities, the established optimization problem is solved efficiently using a gradient-based algorithm. The proposed algorithms are the invention of a novel class of adaptive refinement strategies in isogeometric analysis which surprisingly have not been explored so far.

We study the performance of these algorithms on Poisson equation with either smooth or rapidly varying solution and conduct a convergence rate analysis. The numerical results demonstrate more than one order higher rate of convergence for a sufficiently smooth problem compared to the results of classic NURBS-based IGA. Further, owing to the high capability of rational functions in capturing rapidly varying fields, the method particularly yields remarkably higher accuracy in case of problems containing thin layers and having poor regularity. Overall, the suggested GNURBS-based w-refinement procedures provide a novel powerful adaptive technique in isogeometric analysis and a competitive tool with hierarchical splines.

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ADAPTIVE LEVEL SET XFEM TOPOLOGY OPTIMIZATION WITH HIERARCHICAL B-SPLINES

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ABSTRACT

Topology optimization has become immensely popular in recent years, largely driven by advances in rapid prototyping and additive manufacturing. Topology optimization allows one to optimize material layout within a given design space given a set of loading criteria, boundary conditions, and constraints with the goal of maximizing performance. As opposed to shape optimization, topology optimization allows for topological changes in addition to shape changes. In this talk, I will discuss our recent work on developing an adaptive topology optimization framework based on three component technologies: (i) the level set method, (ii) the extended finite element method, and (iii) hierarchical B-splines. In our framework, the material layout of the domain is defined by one or more level set fields, each of which are defined using hierarchical B-splines. The response of the system is also described by a hierarchical B-spline discretization in conjunction with a generalized version of the extended finite element method. Our framework is general in that one may employ different hierarchical B-spline spaces to describe the level set fields and the system response. Our framework is adaptive in two ways. First, our framework allows for adaptivity of the hierarchical B-spline space describing the system response to better capture local solution features for a given design. Second, our framework allows for adaptivity of the hierarchical B-spline space describing the level set fields to adapt to emerging topological and geometric features as the topology optimization process continues. It should also be noted that since we use hierarchical B-splines to describe the level set fields as opposed to standard finite elements, our framework yields optimal designs which are smooth. I will give an overview of the details of our adaptive topology optimization framework and its properties, and I will provide several examples illustrating its utility in structural and heat conduction topology optimization. I will conclude by briefly discussing a geometric multigrid methodology we have developed to solve the forward and adjoint problems arising in our adaptive topology optimization framework.

A POSTERIORI ERROR ESTIMATION BASED ISOGEOMETRIC METHOD FOR PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

Recently, there has been much progress on the topic of the generalization of splines construction which allow for local refinement, however, an automatic reliable and efficient adaptive refinement routine is still one of the key issues in isogeometric analysis [1]. In order to achieve a fully automatic refinement routine to solve PDEs problem with adaptive isogeometric analysis the a posteriori error estimator is required. A posteriori error estimation in numerical approximation of partial differential equations aims at: (i) give an upper bound on the error of numerical solution, if possible give a guaranteed upper bound; (ii) estimate the error locally and assure that this represents a lower bound for the actual error, up to a multiplicative constant (i.e. efficiency); (iii) assure that the ratio of the estimated error and actual error goes to one, i.e., asymptotic exactness.

Three main techniques of a posteriori estimates in the finite element method have evolved during the last two decades; (a) Explicit residual-based estimators (b) Implicit residual based estimators and (c) Recovery based estimators, see Ainsworth and Oden [2]. In [3] and [4], we have developed a posteriori error estimators for adaptive isogeometric analysis using explicit residual based approach and recovery based approach, respectively. In this talk, we present the a new hybrid approach, by combining the idea from [3] and [4], to design a posteriori error estimators in adaptive isogeometric analysis for elliptic and incompressible flow problems. We also discussed the above three properties for our developed posteriori error estimators. The developed a posteriori based adaptive refinement methodology will be tested on some classical benchmark elliptic and incompressible flow problems. The numerical tests illustrate the optimal convergence rates obtained for the unknown, as well as the effectiveness of the proposed error estimators.

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ISOGEOMETRIC ANALYSIS WITH C^1 -SMOOTH HIERARCHICAL SPLINES ON PLANAR TWO-PATCH GEOMETRIES

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ABSTRACT

In recent years locally refinable spline spaces have been often combined with Isogeometric Analysis to construct adaptive methods for the numerical solution of PDEs. In particular, hierarchical splines [6] are a flexible tool with enough properties to guarantee the optimality of the adaptive method [1,2]. Since the isogeometric framework naturally offers the possibility to employ multipatch geometries and globally C^1 spline spaces defined on them to solve fourth-order PDEs, there is the need for spaces providing such continuity. Several works addressed this issue in the case of tensor-product spaces, with two or more patches [3-5]. In this presentation, we will show how these methods can be combined with the hierarchical construction to obtain global C^1 continuous hierarchical splines on two-patch domains. Examples of isogeometric methods based on these spaces and implemented in the GeoPDEs package will be presented to highlight the features of the construction.

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S-SPLINES: A SIMPLE SURFACE SOLUTION FOR IGA AND CAD

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ABSTRACT

This talk discusses a new surface representation called S-splines. Local refinement of S-spline surfaces is much simpler to understand and to implement than T-spline refinement. Furthermore, no unwanted control points arise in S-spline refinement, unlike T-spline refinement. The refinement algorithm assures linear independence of blending functions. Thus, for IGA, S-spline surfaces provide optimal degrees of freedom during adaptive local refinement. S-splines are compatible with NURBS and T-splines, and can easily be added to existing T-spline implementations.

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B-SPLINES BASED DISCRETIZATION OF TEST SPACES FOR DPG METHOD FOR SINGULAR PERTURBATION PROBLEMS

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ABSTRACT

We propose and investigate the application of B-splines to discretize test spaces for the the discontinuous Petrov-Galerkin (DPG) finite element framework for singular perturbation linear problems, with an emphasis on 2D convection-dominated diffusion. Providing robust L2 error estimates for the field variables is considered a convenient feature for this class of problems. With this requirement in mind, Demkowicz and others [1, 2] have previously formulated special test norms, which through DPG deliver the desired L2 convergence. However, robustness has only been verified through numerical experiments for tailored test norms which are problem-specific, whereas the quasi-optimal test norm (not problem specific) has failed such tests due to the difficulty to resolve the optimal test functions sought in the DPG technology. A deficient resolution of optimal test functions impacts the a-posteriori error estimator and the refinement strategy. With this specific issue in mind (i. e. improving resolution of optimal test functions using the quasi-optimal norm, and thus the refinement process) is that we propose to discretize the local test spaces with B-splines whose knot vectors depend on the perturbation parameter. Two examples are run using adaptive h-refinements and several criteria are analyzed.

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ADAPTIVE ISOGEOMETRIC METHODS AND REDUCED ORDER MODELING

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ABSTRACT

Reduced Order Modelling (ROM) [1] is a popular method to facilitate parameter study or real-time simulation of industrial problems in structural and fluid mechanics. We will herein present and study how well ROM performs for highly graded adaptive grids. Furthermore, we will present a methodology for quality assurance of ROM-simulations. This includes adaptive refinement based on a posteriori error estimation [2,3] of the isogeometric high-fidelity solver together with error estimates of the degradation of the accuracy in a ROM-simulation compared to the related high-fidelity simulation.

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JOINT IMAGE SEGMENTATION AND REGISTRATION BASED ON A DYNAMIC LEVEL SET APPROACH USING TRUNCATED HIERARCHICAL SPLINES

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ABSTRACT

We present an efficient approach for joint image segmentation and nonrigid registration based on a level set formulation. Joint image segmentation and registration is an efficient tool as it incorporates automatic structural analysis into the image processing framework. This method has shown an improved performance as compared to carrying out the segmentation and registration methods separately [1,2].
Unlike previous approaches, the implicit level set function defining the segmentation contour and the spatial transformation that maps the deformation for the image registration are both defined using C^2 continuous hierarchical B-splines. This joint level set framework uses a variational form of an atlas-based segmentation together with a nonrigid registration method which can compute large deformations with cubic splines. The minimization of the variational form is accomplice by dynamic evaluations on a set of successively refined adaptive grids at multiple image resolutions. The improvement in the description of the segmentation result using higher order splines leads to a better accuracy of both the image segmentation and registration process. The performance of the proposed method is demonstrated on 2D synthetic and medical images to show the advantages of the proposed method as compared to other joint segmentation and registration methods.

Keywords: joint image registration and segmentation, adaptive refinement, level set framework, partial differential equation models, hierarchical B-splines, dynamic scheme

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Projection and Transfer Operators in Adaptive Isogeometric Analysis with Hierarchical B-Splines

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ABSTRACT

The finite element discretisation of a large class of boundary value problems requires highly refined meshes to appropriately resolve e.g. singularities in contact problems, shear bands in elasto-plasticity or steep local gradients in the field variables of phase-field models. If these domains evolve during the simulation, adaptive and local mesh refinement and coarsening are essential for efficient computations.

Isogeometric Analysis (IGA) introduced by Hughes et al. [1] overcomes the disjunction between geometry and computational models. Hence, IGA is the ideal discretisation technique to be combined with adaptive mesh refinement as already the coarsest mesh provides an exact geometry representation which is preserved during refinement. Tedious interactions with an underlying geometry during re-meshing, which is required to increase the accuracy of the geometry representation in standard FEM, are avoided.

We present projection methods and transfer operations required for adaptive mesh refinement/coarsening in problems with internal variables that possess own evolution equations at integration point level. By extending the results of Hennig et al. [2], we propose three different local and semi-local least squares projection methods for field variables and compare them to the standard global version. We discuss the application of two different transfer operators for internal variables. An alternative new operator inspired by superconvergent patch recovery [3] is also proposed.

The presented projection methods and transfer operations are tested in benchmark problems and applied to phase-field modelling of spinodal decomposition and brittle and ductile fracture.

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LOW RANK PARAMETRIZATION OF SOLIDS

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ABSTRACT

Matrix assembly for isogeometric discretization of partial differential equations remains a challenging process. Storage requirements and computation times are indeed higher than in classical finite elements due to a larger support of the basis functions. It results a restriction of the number the degrees of freedom that can be used. An alternative approach to adaptivity, proposed in [1], permits to improve drastically the overall efficiency of matrix assembly using the global tensor-product structure of the spline bases to decompose the matrix. The efficiency of this method then strongly depends on the parametrization's rank of the considered geometry. A low rank interpolation of spline curves have then been proposed in [2]. We will present in this talk some new results on solid's low rank parametrization.

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