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ISOGEOMETRIC COLLOCATION IN ACOUSTICS

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ABSTRACT

In this work we show some applications of isogeometric collocation to problems of two- and three-dimensional acoustic scattering and radiation, modeled by the Helmholtz equation in an unbounded domain. The two major difficulties of numerical methods in acoustics are associated with a) a highly oscillatory nature of the solutions as the wave number ‘k’ increases, and b) the necessity to truncate the infinite domain with an artificial surface, where the Sommerfeld radiation condition is approximated.

Isogeometric collocation can successfully address both these difficulties since

(a) it requires less computational time in comparison with Galerkin methods and therefore can handle more DOFs in feasible times to capture the oscillatory behavior of solutions. Moreover, the pollution error, associated with large values of ‘k’ can be significantly reduced if higher degree approximations are used, making ‘p’-refinement very efficient.
(b) the boundary conditions on the truncation surface involve higher order derivatives along the boundary and can be efficiently handled with the NURBS parameterizations. Moreover, some of the advanced boundary conditions require approximating a family of special boundary functions, which can also be discretized by the same NURBS, as used for the solution, making the overall model homogeneous and consistent.

We demonstrate the application of the method on some benchmark problems. We perform a detailed analysis of the truncation error for four different boundary conditions. Numerical studies indicate the efficiency of the approach.

ISOGEOMETRIC SIMULATION BASED ON NURBS COLLOCATION AND UNIFIED SPACE-TIME FORMULATION

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ABSTRACT

Isogeometric Analysis (IGA) was first introduced in 2005 as an alternative method for finite element analysis (FEA) by integrating Computer Aided Design (CAD) and downstream analysis without the use of an intermediate mesh model [1]. Most researches on isogeometric analysis are based on either the Galerkin (IGA-G) or collocation (IGA-C) formulations. Isogeometric collocation methods provide the potential to increase the computational efficiency of IGA with a specified level of accuracy [2]. Space-time isogeometric analysis (ST-IGA) is also an important area in IGA for producing time-dependent solutions with given boundary and initial conditions. Commonly used formulation in space-time isogeometric analysis is mostly based on time-stepping methods [3, 4, 5]. In this study, we present a method for ST-IGA based on NURBS collocation and unified space-time formulation. Both spatial and time spaces are integrated as the single extended computation space. The computation and the solution spaces are further represented using the same NURBS formulation. Collocation methods are finally applied for producing the IGA solutions. We have applied the proposed method for the simulation of temperature distribution in thermal analysis. For simplicity, we restrict ourselves to fixed spatial computational domains in the present study, but the method can also be extended to solutions over a variable spatial domain. Numerical examples show that the resulting solutions are stable with standard convergence rates as that of collocation methods in both the spatial and temporal dimensions.

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Mixed isogeometric collocation methods for incompressible elasticity and poromechanics

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Isogeometric Analysis (IGA) is a recent analysis framework (cf. [1,2]) aiming at bridging the gap between Computational Mechanics and Computer Aided Design (CAD). In addition to clear advantages in terms of geometry representation capabilities, the use of functions typically used by CAD systems (e.g., NURBS) leads to superior results with respect to standard finite elements on a per degree-of-freedom basis, thanks to their high regularity properties.

In the framework of NURBS-based IGA, collocation methods have been recently introduced as an efficient and promising alternative to standard isogeometric Galerkin approaches (cf. [3]), characterized by a high accuracy-to-computational-cost ratio (see [4]). In this work, we study the approximation of incompressible elastic problems via isogeometric collocation. In particular, we introduce and discuss a mixed u-p formulation and we present a number of numerical tests showing the behavior of the proposed method. Moreover, we initiate the study of deformable fluid-saturated porous media. The combination of the superior accuracy and smoothness of spline basis functions with the low computational cost and simplicity of collocation techniques seems to constitute an optimal basis for accurately modeling complex and computationally demanding time-dependent problems expressed in mixed form, like those arising in the context of poroelastic media. In particular, we will focus on the one-dimensional applications of the Biot model and present a mixed *u-p* formulation leading to very encouraging results.

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REDUCED INTEGRATION AT SUPERCONVERGENT POINTS IN ISOGEOMETRIC ANALYSIS

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ABSTRACT

In this talk, I will present a new reduced integration rule [1] for isogeometric analysis (IGA) based on the concept of variational collocation [2]. It has been recently shown that, if a discrete space is constructed by smooth and pointwise non-negative basis functions, there exists a set of points – named Cauchy–Galerkin (CG) points – such that collocation performed at these points can reproduce the Galerkin solution of boundary value problems exactly. Since CG points are not known a-priori, estimates are necessary in practice and can be found based on superconvergence theory [2], [3], [4]. I will show how estimated CG points (i.e. superconvergent points) can be used as numerical quadrature points to obtain an efficient and stable reduced quadrature rule in IGA. We use the weighted residual formulation (without integration by parts) as basis for our new quadrature rule, so that the proposed approach pertains neither to standard Galerkin variational formulations nor to direct collocation of the strong form. The performance of the method is demonstrated by several examples. For odd degrees of discretization, we obtain spatial convergence rates and accuracy very close to those of accurately integrated standard Galerkin with a quadrature rule of two points per parametric direction independently of the degree.

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