

# ADVANTAGES OF IDENTICAL SPLINE-BASED INTERFACES FOR FLUID-STRUCTURE INTERACTION

Norbert Hosters<sup>1</sup>, Michel Make<sup>1</sup>, Thomas Spenke, Stefanie Elgeti<sup>1</sup> and Marek Behr<sup>1</sup>

<sup>1</sup> Chair for Computational Analysis of Technical Systems (CATS), CCES,  
RWTH Aachen University, 52062 Aachen, Germany  
{hosters,make,spenke,elgeti,behr}@cats.rwth-aachen.de

## ABSTRACT

The introduction of isogeometric analysis (IGA) [1], made it possible to directly exploit the favorable geometric properties of NURBS for numerical analysis. The method has become widespread in structural mechanics. However, parametrizing complex three-dimensional domains – as needed for CFD – using closed volume splines can be challenging. NURBS-enhanced finite elements (NEFEM) [2] can be a viable alternative. Both methods together lead to a geometrically compatible coupling interface for FSI. Thus, the geometric gap between the discretizations is closed. Within a partitioned FSI method, it was demonstrated that the necessary projection methods simplify due to the matching geometry; while at the same time increasing accuracy [3].

In the current work, we apply the described approach to FSI problems involving incompressible and compressible flows. In these problems, accurate geometric representation can be important, e.g., in case of enclosed domains, free-surface flows or due to the presence of shock waves and their interaction with solid walls. Advantages of the proposed methods are demonstrated on several test cases. For this purpose, a partitioned approach is applied. The incompressible or compressible Navier-Stokes equations are solved by the Deforming Spatial Domain/Stabilized Space-Time (DSD/SST) procedure extended by NEFEM. The elastodynamics problem is solved by means of NURBS-based isogeometric analysis. Results are compared against solutions obtained with a standard finite element formulation.

## REFERENCES

- [1] T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs, Isogeometric analysis: CAD, Finite Elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, **194**, 4135-4195, 2005.
- [2] R. Sevilla; S. Fernandez-Mendez and A. Huerta . NURBS-Enhanced Finite Element Method (NEFEM): A Seamless Bridge Between CAD and FEM. *Arch of Computat. Methods Eng.*, **18**, 441, 2011.
- [3] N. Hosters, J. Helmig, A. Stavrev, M. Behr and S. Elgeti, Fluid-Structure Interaction with NURBS-based Coupling, *Comput. Methods Appl. Mech. Engrg.* (2018)

# ISOGEOMETRIC ANALYSIS FOR COMPRESSIBLE FLOWS

Matthias Möller<sup>1</sup>, and Andrzej Jaeschke<sup>2</sup>

<sup>1</sup>Delft University of Technology, Department of Applied Mathematics,  
Van Mourik Broekmanweg 6, 2628XE Delft, The Netherlands  
m.moller@tudelft.nl

<sup>2</sup>Lodz University of Technology, Institute of Turbomachinery,  
ul. Wolczanska 219/223, 90-924 Lodz, Poland  
andrzej.jaeschke@p.lodz.pl

## ABSTRACT

An isogeometric analysis framework for simulating compressible flows is presented and its HPC implementation in the open-source software G+Smo ([www.gs.jku.at/gismo](http://www.gs.jku.at/gismo)) is described.

The framework is designed with heterogeneous hardware platforms in mind. The overall design philosophy is to associate individual compute devices like multi-core CPUs and many-core GPUs with individual patches of the multi-patch IGA parameterization. A pre-processing tool for screw-compressor geometries has been developed in [1], which ensures that each single patch provides sufficient computational work to use the compute capabilities of the device to full capacity. This also includes the possibility to assign fully structured tensor-product discretizations with many degrees of freedom to GPUs and keep THB-refined patches with possibly less DOFs on CPUs.

Modern meta-programming techniques are adopted to combine different hardware-optimized linear algebra back-ends with the fluid dynamics expression-template library FDBB [2]. This enables us to implement the core components of the flow solver as hardware independent compute kernels, which get just-in-time compiled and thereby optimized for each individual patch at run time.

Mathematically, the flow solver is based on a generalization [3] of the Algebraic Flux Correction paradigm [4] to higher-order B-Spline spatial discretizations and adopts strong stability preserving explicit Runge-Kutta time-integration schemes for advancing the solution forward in time.

## REFERENCES

- [1] J. Hinz, M. Möller, and K. Vuik, Elliptic Grid Generation Techniques in the Framework of Isogeometric Analysis Applications, accepted for publication in *Computer Aided Geometric Design*, DOI: [10.1016/j.cagd.2018.03.023](https://doi.org/10.1016/j.cagd.2018.03.023).
- [2] M. Möller: FDBB: Fluid dynamics building blocks, in: *Proceedings of the 6th European Conference on Computational Mechanics (ECCM 6), 7th European Conference on Computational Fluid Dynamics (ECFD 7)*, 11<sup>th</sup>-15<sup>th</sup> June 2018, Glasgow, United Kingdom.
- [3] M. Möller and A. Jaeschke, High-Order Isogeometric Methods for Compressible Flows. II. Compressible Euler Equations, in: *Proceedings of the 19th International Conference on Finite Elements in Flow Problems (FEF 2017)*, 5<sup>th</sup>-9<sup>th</sup> April 2018, Rome, Italy.
- [4] D. Kuzmin, M. Möller, and M. Gurrus, Algebraic flux correction II. Compressible flow problems, in: Kuzmin et al. (eds.) *Flux-Corrected Transport: Principles, Algorithms, and Applications*, pp. 193–238. Springer, 2nd edition, 2012.

# ISOGEOMETRIC MODELLING OF CELLULAR MEMBRANES

Roger A. Sauer<sup>1</sup>, Amaresh Sahu<sup>2</sup>, Yannick A.D. Omar<sup>1</sup> and Kranthi K. Mandadapu<sup>2</sup>

<sup>1</sup>Graduate School AICES, RWTH Aachen University, Aachen, Germany  
sauer@aices.rwth-aachen.de, omar@aices.rwth-aachen.de

<sup>2</sup>Department of Chemical & Biomolecular Engineering, University of California at Berkeley, USA  
amaresh.sahu@berkeley.edu, kranthi@berkeley.edu

## ABSTRACT

This work discusses the computational modeling of lipid bilayer membranes based on the nonlinear theory of thin shells [1]. Several computational challenges are identified and various theoretical and computational ingredients are proposed in order to counter them [2]. In particular, C1-continuous, NURBS-based, LBB-conforming surface finite element discretizations are discussed [3]. The constitutive behavior of the bilayer is based on in-plane viscosity and (near) area-incompressibility combined with the elastic Helfrich bending model. The behavior is thus fluid-like in-plane and solid-like out-of-plane, leading to a coupled fluid-structure interaction problem. For quasi-static computations those require various shear stabilization techniques. For dynamic computations the implicit Euler scheme is used for the temporal discretization. Further, a new arbitrary Lagrangian-Eulerian surface formulation is proposed to handle general surface flows. All ingredients are formulated in the curvilinear coordinate system characterizing general surface parametrizations. Those allow for an extension of the formulation to thermal and chemical coupling [4]. Several numerical examples are presented that exhibit large membrane shape changes involving non-trivial surface flows.

## REFERENCES

- [1] R.A. Sauer and T.X. Duong, On the theoretical foundations of thin solid and liquid shells, *Math. Mech. Solids*, **22**, 343-371, 2017.
- [2] R.A. Sauer, On the computational modeling of lipid bilayers using thin-shell theory, in D. Steigmann (Ed.), *The role of mechanics in the study of lipid bilayers*, Springer, 2018.
- [3] R.A. Sauer, T.X. Duong, K.K. Mandadapu and D.J. Steigmann, A stabilized finite element formulation for liquid shells and its application to lipid bilayers, *J. Comput. Phys.*, **330**, 436-466, 2017.
- [4] A. Sahu, R.A. Sauer and K.K. Mandadapu, The irreversible thermodynamics of curved lipid membranes, *Phys. Rev. E*, **96**, 042409, 2017.

# ON THE COMBINATION OF MORTAR CONTACT METHODS WITH ISOGEOMETRIC DISCRETIZATION

Thang Xuan Duong<sup>1</sup>, Laura De Lorenzis<sup>2</sup>, and Roger A. Sauer<sup>3</sup>

<sup>1</sup> Aachen Institute for Advanced Study in Computational Engineering Science (AICES),  
RWTH Aachen University, Germany  
duong@aices.rwth-aachen.de

<sup>2</sup> Institute of Applied Mechanics, Technische Universität at Braunschweig, Germany  
l.delorenzis@tu-braunschweig.de

<sup>3</sup> Aachen Institute for Advanced Study in Computational Engineering Science (AICES),  
RWTH Aachen University, Germany  
sauer@aices.rwth-aachen.de

## ABSTRACT

Non-uniform rational basis spline (NURBS) has provided a promising interpolation method for solving contact problems using finite element methods [1]. This is mainly due to its advantage in the smooth and accurate description of contact surfaces with coarse meshes, which may enhance both the accuracy and the robustness of contact formulations. On the other hand, mortar contact methods [2] improve the robustness by enforcing contact constraints weakly. Mortar formulations differ on the choice of either mortar shape functions to interpolate the contact pressure [3], or work conjugation pairs [4], which can vary the degree of the weakness of the constraint enforcement in mortar methods. We therefore concern ourselves with benefits from the NURBS discretizations on mortar methods in the context of arbitrarily large deformation and large sliding contact.

In this contribution, we compare the performance - in terms of the accuracy and robustness - of various mortar formulations (see [4]) by mean of several challenging numerical examples, when combined with NURBS discretizations. The comparison aims at finding suitable choices of the mortar shape functions and work conjugation pairs in order to benefit from NURBS in mortar methods. The penalty regularization for both frictional and frictionless contact is considered.

## REFERENCES

- [1] De Lorenzis, L., Wriggers, P., and Hughes, T. J. R. Isogeometric contact: A review. *GAMM Mitteilungen*. **37**, 85-123, 2014.
- [2] Puso, M. A. and Laursen, T. A. A mortar segment-to-segment contact method for large deformation solid mechanics. *Comput. Methods Appl. Mech. Engrg.* **193**, 601-629, 2004.
- [3] Duong, X. T., Lorenzis, L. D., and Sauer, R. A. On the shape functions for the contact pressure in mortar methods. In von Scheven, M., Keip, M.-A., and Karajan, N., editors, *Proceedings of the 7th GACM Colloquium on Computational Mechanics*, 130-133, 2017.
- [4] Duong, X. T., De Lorenzis, L., and Sauer, R. A. A segmentation-free isogeometric extended mortar contact method. *Computational Mechanics*, **accepted**, 2018.

# PDE-BASED PARAMETERIZATION TECHNIQUES FOR IGA-APPLICATIONS

J. Hinz<sup>1\*</sup>, M. Möller<sup>1</sup>, and C. Vuik<sup>1</sup>

<sup>1</sup>Delft University of Technology, Department of Applied Mathematics, Van Mourik  
Broekmanweg 6, 2628 XE Delft, the Netherlands  
{j.p.hinz, m.moller, c.vuik}@tudelft.nl

## ABSTRACT

Isogeometric Analysis (IgA) [1] has become an accepted numerical technique for the simulation of engineering processes. However, the fully automated generation of analysis-suitable parameterizations of geometries as they arise in practical work flows is still a challenging task which often requires application-specific approaches.

In this talk we adopt the principles of ‘*Elliptic Grid Generation*’ [2] for the PDE-based generation of planar parameterizations for various challenging geometries starting from a description of their boundary contours. We present several numerical techniques to approximately solve the resulting nonlinear PDE-equations along with automated reparameterization techniques that warrant a high-quality parameterization for geometries subject to extreme aspect ratios.

Numerical experiments with twin-screw compressor geometries demonstrate the efficiency and reliability of the proposed approaches since an analysis-suitable planar parameterization is typically achieved within 3-4 nonlinear iterations. In settings that require a large number of slightly varied parameterizations of the same base geometry (such as swept-volume and shape-optimization applications), the efficiency is further improved by a database-driven approach.

The low cost-to-quality ratio of the proposed techniques make them a viable tool to fill the gap between the computationally inexpensive algebraic methods and the expensive (constrained) grid quality functional minimization, since the latter can be initialized with the folding-free PDE-solution, while the former is initialized with an algebraic initial guess.

## REFERENCES

- [1] Hughes, Thomas JR, John A. Cottrell, and Yuri Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, *Computer methods in applied mechanics and engineering* 194.39-41 (2005): 4135-4195.
- [2] J. Hinz, M. Möller, C. Vuik, Elliptic grid generation techniques in the framework of isogeometric analysis applications, *Accepted for publication in: Computer Aided Geometric Design (2018)*, DOI: 10.1016/j.cagd.2018.03.023

## Weak $C^1$ dual mortar method and its application for Kirchhoff-Love shell

Di Miao<sup>1</sup>, Michael J. Borden<sup>2</sup>

<sup>1</sup> Civil & Environmental Engineering  
Brigham Young University  
Provo, UT 84602, USA  
miaodi1987@gmail.com

<sup>2</sup> Civil & Environmental Engineering  
Brigham Young University  
Provo, UT 84602, USA  
mjborden@byu.edu

### ABSTRACT

Due to the tensor product structure, conventional non-uniform rational B-splines (NURBS) surfaces become inefficient when dealing with complex geometries. Though multi-patch NURBS can be used to discretize complicated domain, additional treatments are needed to handle non-conforming coupling along patch interfaces. Among popular domain decomposition methods, mortar method is superior in that it neither introduces additional degrees of freedom nor requires the evaluation of penalty parameters. We propose a weak  $C^1$  dual mortar method, which can be used to tackle 4<sup>th</sup> order PDEs. Owing to the locality of Bezier dual basis, the constrained system remains sparse and can be solved in an efficient manner. We also propose a new algorithm to treat cross-points without the need to modify boundary element. The optimality of the proposed method has been confirmed in biharmonic problems and Kirchhoff-Love shell problems by several numerical examples.